AUGUST 1978 PPPL-1471
UC-20g

ALFVÉN RESONANCE HEATING VIA MAGNETOSONIC MODES IN LARGE TOKAMAKS

BY

C. F. F. KARNEY AND F. W. PERKINS

PLASMA PHYSICS LABORATORY

*

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

This work was supported by the U. S. Department of Energy Contract No. EY-76-C-02-3073.

Alfvén Resonance Heating Via Magnetosonic Modes in Large Tokamaks

Charles F. F. Karney and Francis W. Perkins
Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08540

ABSTRACT

The theory of Alfvén resonance heating of tokamaks is extended beyond the incompressible MHD model to include finite ω/Ω_i effects, which lead to off-diagonal terms in the conductivity tensor, and compressibility, which permits the fast Alfvén mode. The finite frequency effects can greatly change the dissipation resulting from the shear Alfvén resonance. With an appropriate choice of parameters, the dissipation can vanish allowing high-Q toroidal eigenmodes in large tokamaks such as PLT.

Tokamaks of the size of PLT are the first to nominally permit propagation of magnetosonic (i.e., compressional Alfvén) modes at frequencies that are a modest fraction of the ion-cyclotron frequency. (Typically $\omega/\Omega_i \simeq 0.5$.)

In this frequency range, the transition from the high density propagation region for magnetosonic waves to the low density evanescent region does not occur via a simple cutoff, but rather by a closely-spaced cutoff-resonance-cutoff triplet. The resonance, which occurs at the point in the plasma profile where $\omega \approx k_{||}v_A$, is called the shear Alfvén resonance and generally leads to energy absorption. The goal of this work is to evaluate how much absorption the shear Alfvén resonance causes. We accomplish this by calculating the reflection coefficient for magnetosonic waves incident on the cutoff-resonance-cutoff triplet.

Previous analyses of the problem[1,2] treated the MHD limit in which $\omega/\Omega_i \to 0$, and, by and large, ignored compressibility thus suppressing magnetosonic modes. The recent work of Ott, et al.,[2] does recognize that the shear Alfvén resonance can damp magnetosonic modes but does not include finite frequency effects, which we find to be important, and specializes its results to the m = 0 ($k_y = 0$) case.

Since the dissipation depends on the fields close to the region of the cutoffs and resonance, which typically occupy only a small fraction of the minor radius of a tokamak, we may accurately treat the problem by a slab geometry model in which x is parallel to the density gradient and z is parallel to the constant magnetic field, B_o. A linear variation of density with x is employed. The neglect of the shear of the magnetic field is justifiable for a tokamak as long as $n >> m/q$ (n and m are the toroidal and poloidal mode numbers and q is the safety factor). The electrons and ions are treated as separate cold fluids. All field quantities vary as $f(x)\exp(ik_y y + ik_z z - i\omega t)$. The parallel electrical field is neglected (it is shorted out by the high parallel electron conductivity). The remaining components of the electric field are described by

$$(A - k_y^2)\, E_x + i(D - k_y d/dx)E_y = 0\,, \qquad (1)$$

$$-i(D + k_y d/dx)E_x + (A + d^2/dx^2)E_y = 0\,, \qquad (2)$$

where $A = (\omega^2/v_A^2)[\Omega_i^2/(\Omega_i^2 - \omega^2)] - k_z^2$, $D = (A + k_z^2)\omega/\Omega_i$ and $v_A = c\Omega_i/\omega_{pi}$. Making the substitutions $E = E_y$ and

$$\psi = (A - k_y^2)^{-1}(A\, d/dx - k_y D)E_y\,, \qquad (3)$$

we may recast (1) and (2) as a coupled set of first order differential equations,

$$\frac{d}{dx}E - \frac{k_y D}{A}E = \frac{A - k_y^2}{A}\psi \qquad (4)$$

$$\frac{d}{dx}\psi + \frac{k_y D}{A}\psi = -\left(\frac{A^2 - D^2}{A}\right)E \qquad (5)$$

The right-hand side of Eq. (5) is the cutoff-resonance-cutoff triplet.

We put Eqs. (4) and (5) into dimensionless form by defining a scalelength $\ell = [A'(x_o)]^{-1/3}$ and a new coordinate $\xi = (x - x_o)/\ell$, where $A' = dA/dx$ and x_o is the point at which $A = 0$ (the finite frequency generalization of the shear Alfvén resonance). For small ξ we retain only the leading order terms in the series expansions, for A and D. Thus $A \simeq \xi/\ell^2$ and $D \simeq S/\ell^2$, where $S = k_{||}^2 \ell^2 (\omega/\Omega_i)$. Equations (4) and (5) then become

$$\frac{d}{d\xi} E - \frac{MS}{\xi} E - \frac{\xi - M^2}{\xi} \Psi = 0 \tag{6}$$

$$\frac{d}{d\xi} \Psi + \frac{MS}{\xi} \Psi + \frac{\xi^2 - S^2}{\xi} E = 0 \; . \tag{7}$$

where $M = k_y \ell$ and $\Psi = \ell\psi$. The equation for E reads

$$\frac{d^2}{d\xi^2} E - \frac{M^2}{\xi(\xi - M^2)} \frac{d}{d\xi} E + \left(\frac{\xi^2 - S^2}{\xi} - M^2 + \frac{MS}{\xi(\xi - M^2)} \right) E = 0 \; . \tag{8}$$

We note the following points about Eq. (8). The equation is singular at $\xi = 0$, the shear Alfvén resonance. The point $\xi = M^2$ is only an apparent singularity as may be seen from Eqs. (6) and (7). This point is the cutoff for the magnetosonic wave. In the limit $|\xi| \to \infty$ the equation becomes the Airy equation, $d^2E/d\xi^2 + (\xi - M^2)E = 0$, which physically describes a propagating magnetosonic wave for $\xi > M^2$ and an evanescent mode for $\xi < M^2$.

We wish to determine the reflection coefficient, for this will determine Q of magnetosonic modes. To do this we solve Eq. (8) with the boundary condition that as $\xi \to -\infty$, $E \propto \mathrm{Ai}\,(M^2 - \xi)$, the evanescent Airy function. We ignore the small portion of the Bi function which should be added since the boundary of the plasma where $E = 0$ is at the finite distance $\xi = -x_o/\ell$. By integrating Eq. (8) through the Alfvén resonance layer we find what combination of the Ai and Bi functions we have at $\xi \to \infty$. (Treating the frequency as a Laplace transform variable, we find that causality requires us to go above the singularity at $\xi = 0$ in the complex ξ plane.) We decompose the Ai and Bi functions into a wave incident on the layer $\xi = M^2$ and a reflected wave. We define an amplitude reflection coefficient, R, and from it the fraction, q, of incident power dissipated upon reflection $q = 1 - |R|^2$.

In cases of interest, S and M are small. An analytical estimate of q is then possible. We obtain this by finding series solutions to Eq. (8) about the singular point and by matching these onto the Airy functions at large ξ. In the neighborhood of the singularities, the solutions are (to lowest orders in M and S)

$$E_1 = 1 + \left(\frac{S}{M} + S^2\right)\xi + O(S^4, \xi^3, S^2\xi^2), \tag{9}$$

$$E_2 = 1 + MS\,(1 - MS)\,E_1 \log(\xi) + O(S^4, \xi^3, S^2\xi^2). \tag{10}$$

[We assume $M = O(S)$.] The appropriate combination of E_1 and E_2 that matches onto $Ai(-\xi)$ in the region $M^2 << -\xi \sim \xi_o << 1$ is

$$E = [c_1 - (M/S)c_2]E_2 + [(M/S)c_2 + \log(-\xi_o)(M^2c_2 - MSc_1)]E_1, \quad (11)$$

where $c_1 = Ai(0) = 0.355$ and $c_2 = -Ai'(0) = 0.259$. We write $\log(-\xi_o) = \log|\xi_o| + i\pi$, and then ignore the $\log|\xi_o|$ term since it is smaller than the other real terms. For $M^2 << \xi << 1$ we then have

$$E = Ai(-\xi) + i(\pi/2\sqrt{3})[M(c_2/c_1)^{1/2} - S(c_1/c_2)^{1/2}]^2 \, Bi(-\xi), \quad (12)$$

from which we determine the fractional power absorbed,

$$q = (2\pi/\sqrt{3})[M(c_2/c_1)^{1/2} - S(c_1/c_2)^{1/2}]^2 . \quad (13)$$

The MHD limit is given by $S \to 0$. We see that the effect of the finite frequency is either to increase or to decrease q depending on the sign of M. If $S = Mc_2/c_1$ we have $q = 0$.

In Fig. 1 we compare Eq. (13) with the value of q obtained by numerically integrating Eqs. (6) and (7). We see that there is good agreement for $|S|, |M| \lesssim 0.5$. In particular the line along which $q = 0$ is accurately given by Eq. (13).

To illustrate our result let us compute q for PLT taking $k_y = -m/a$, $k_{||} = n/R$, $a = 40\text{cm}$, $R = 1.3\text{m}$ and assuming a parabolic density profile. We find

$$q = 0.1\, m^2 \left(\frac{B_{40}}{a_{40}\, f_{25}}\right)^{4/3} \left(\frac{1}{n_{14}}\right)^{2/3} \left\{ 1 + \left(\frac{a}{R}\right)^2 \frac{n^2}{10m} \left[\frac{f_{25}}{B_{40}\, n_{14}\, a_{40}^2}\right]^{1/3} \right\}^2 \tag{14}$$

where B_{40} is the toroidal field in units of 40kG, f_{25} is the frequency of the wave generator in units of 25 MHz, n_{14} is the central density in units of $10^{14}\,cm^{-3}$ and a_{40} the minor radius in units of 40cm. Thus if $m = -1$, q is small for $n \simeq \pm 10$. With this value of n the wave energy propagates away from the antenna reasonably quickly (since $k_\perp \sim k_\parallel$) so that the parallel damping length is long. We expect, therefore, that a high-Q eigenmode will be excited. Practically, this allows us to combine the high antenna loading associated with toroidal eigenmodes[3] with the Alfvén wave dissipation process.

The physics of the dissipation mechanism has not been discussed here. Inclusion of finite temperature and parallel electric field effects[4] shows that the dissipated energy is linearly mode-converted[5] into a thermal wave which propagates into the plasma from the resonance. This wave damps via linear electron Landau damping. Hence the energy absorbed by the shear Alfvén resonance shows up as electron heating near the resonance.

This work was supported by the U. S. Department of Energy Contract No. EY-76-C-02-3073.

REFERENCES

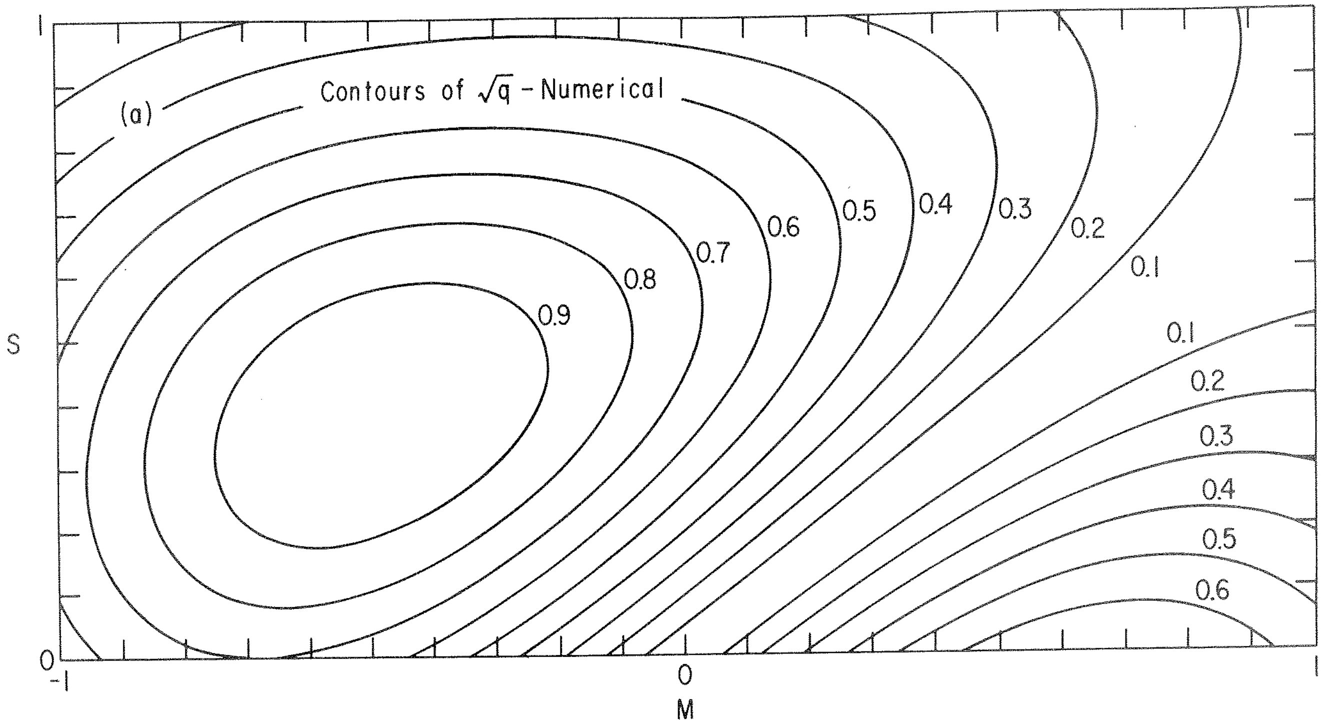
[1]L. Chen and A. Hasegawa, Phys. Fluids 17, 1399 (1974);
J. A. Tataronis and W. Grossman, Nucl. Fusion 16, 667 (1976);
J. A. Tataronis, J. Plasma Phys. 13, 87 (1975);
J. M. Kappraff and J. A. Tataronis, J. Plasma Phys. 18, 209 (1977).

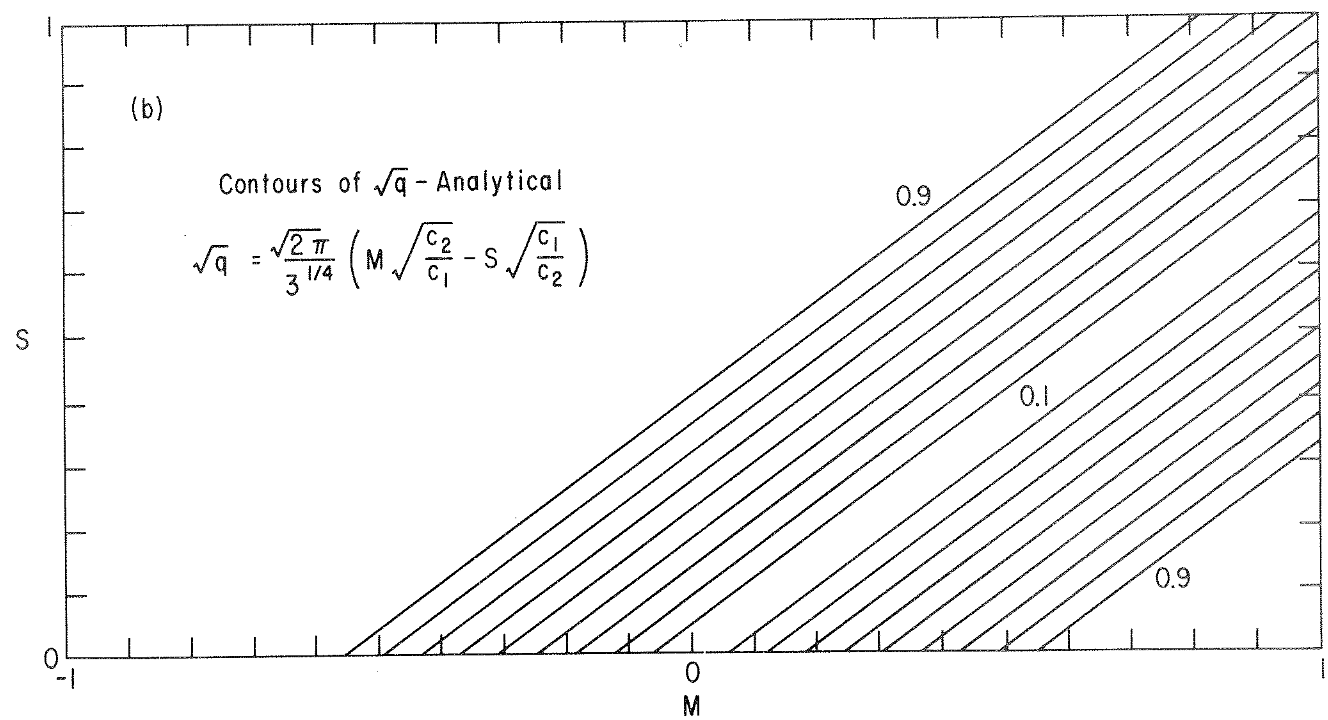
[2]E. Ott, J.-M. Wersinger, and P. T. Bonoli, Laboratory of Plasma Studies Report LPS 236, Cornell University (March, 1978).

[3]J. Adam et al., in Plasma Physics and Controlled Nuclear Fusion Research 1974 (IAEA, Vienna, 1975) p. 65.

[4]A. Hasegawa and L. Chen, Phys. Fluids 19, 1924 (1976).

[5]T. H. Stix, Phys. Rev Lett. 15, 878 (1965).





782194

Fig. 1. Contours of constant $\sqrt{q}$. (a) Results obtained by numerically integrating (6) and (7). (b) Analytic result, (13).