TWO-DIMENSIONAL DEPLETION OF LOWER HYBRID PUMP BY QUASIMODE EXCITATIONS

PRR # 77/19
August 1977

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Submitted to Physics of Fluids.

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ABSTRACT

We study the effect of parametric excitation of quasimodes on the depletion of a lower hybrid pump wave driven from a finite extent source with parameters relevant to tokamak plasmas. The set of coupled nonlinear equations for the pump wave and the sideband mode reduces to those corresponding to the three wave resonant equations with one wave heavily damped. Exact two-dimensional analytic solutions of these equations are investigated in detail to elucidate the effects of the pump width and the convective losses arising out of the finite group velocities of the modes. We also study the scaling of depletion with variation in plasma parameters.

I. INTRODUCTION

In supplementary heating of tokamaks with RF power near the lower hybrid frequency, the RF excitation originates from finite-extent source fields[1,2] (waveguide arrays or slow wave structures) at the wall of the plasma chamber. Nonlinear effects in propagation, such as self-modulation[3] and parametric excitations,[4] can effect the penetration of RF energy into the plasma. In particular, self-modulation can lead to filamentation and subsequent localized heating at the edge, whereas parametric interactions can lead to the redistribution of the lower hybrid energy into other modes or quasimodes. The resultant pump depletion could seriously jeopardize bulk heating of the plasma.

In this paper we make a detailed investigation of the effect of parametric excitation of quasimodes on the depletion of a lower hybrid pump. As earlier linear theory studies have shown,[5] excitation of quasimodes is an important decay process, particularly in the region where $\omega_0 \leq \sqrt{2}\,\omega_{LH}$ and it can have large growth rates for moderate pump powers. We investigate the nonlinear stage of these instabilities using exact analytic solutions to the coupled nonlinear equations with particular emphasis on the two-dimensional aspects of the solutions and geometrical limitations arising out of the finite size of the source.

II. NONLINEAR EQUATIONS AND THEIR SOLUTION

The equations governing this parametric process are straightforward to derive.[6] Basically, we are considering the coupling of the electrostatic pump wave potential $\phi_0(x)\exp(-i\omega_0 t)$ with another lower hybrid wave potential $\phi_1(x)\exp(-i\omega_1 t)$ such that the beat wave with $\omega = \omega_0 - \omega_1$ and $k = k_0 - k_1$ has a phase velocity close to the electron thermal velocity and is consequently heavily damped. The electron density fluctuations corresponding to this low frequency beat mode provide the necessary nonlinear coupling between ϕ_0 and ϕ_1 and the coupled equations can be written in the form:

$$(\nabla_\perp \cdot K_{\perp,0}\nabla_\perp + \frac{\partial}{\partial z} K_{\parallel,0} \frac{\partial}{\partial z}) \phi_0 = \frac{e}{i\omega_0\epsilon_0} \nabla \cdot (n_e \mathbf{v}_1) \tag{1}$$

$$(\nabla_\perp \cdot K_{\perp,1}\nabla_\perp + \frac{\partial}{\partial z} K_{\parallel,1} \frac{\partial}{\partial z}) \phi_1 = \frac{e}{i\omega_0\epsilon_0} \nabla \cdot (n_e^* \mathbf{v}_1) \tag{2}$$

where $K_{\perp,j} = [1 + (\omega_{pe}^2/\Omega_e^2) - (\omega_{pi}^2/\omega_j^2)]$, $K_{\parallel,j} = [1 - (\omega_{pe}^2/\omega_j^2)]$ are components of the cold plasma dielectric function and the other notations are standard. The low frequency density fluctuations are driven by the nonlinear ponderomotive force arising out of the beating of the pump and sideband modes and can be shown to obey the relation

$$n_e = \frac{\epsilon_0}{e} k^2 \frac{(1 + \chi_i)\chi_e}{1 + \chi_i + \chi_e} \phi_p \tag{3}$$

where ϕ_p is the ponderomotive potential given by

$$e(\partial\phi_p/\partial z) = -me[(\mathbf{v}_0 \cdot \nabla)v_{1z}^* + (\mathbf{v}_1 \cdot \nabla)v_{0z}^*] \tag{4}$$

χ_i, χ_e are the ion and electron susceptibilities of the quasimode and account for the self-consistent electrostatic potential at the low frequency. Equation (4) is easily solved for ϕ_p by substituting for the high frequency electron velocities

$$\mathbf{v}_{ej} = -c \frac{\nabla\phi_j \times \hat{z}}{B_0} - \frac{imc^2}{eB_0^2} \omega_j \nabla\phi_j - \frac{ie}{m\omega_j} \nabla_z \phi \, \hat{z} \, . \tag{5}$$

The expression for ϕ_p contains three terms, which are respectively contributions from the $\mathbf{E} \times \mathbf{B}_0$ drift, the polarization drift, and the parallel motion of the electrons. In most cases the dominant contribution is the $\mathbf{E} \times \mathbf{B}_0$ term, which is

$$\phi_p = \frac{-ic}{\omega_j} \left(\frac{\nabla_\perp \phi_1^* \times \nabla_\perp \phi_0 \cdot \hat{z}}{B_0} \right) . \tag{6}$$

Next, we make the usual WKB approximation of

$$\phi_j = \tilde{\phi}_j(x, z) \exp(i\mathbf{k}_j \cdot \mathbf{x}) \tag{7}$$

$$(\phi, \phi_p, n_e) = [\tilde{\phi}(x, z), \tilde{\phi}_p(x, z), \tilde{n}_e(x, z)] \exp(i\mathbf{k} \cdot \mathbf{x}) \tag{8}$$

where the quantities with tilde represent slowly varying envelopes in the x-z plane. Substituting for the various quantities in (1) and (2) it is now straightforward to obtain the following set of coupled equations:

$$\frac{\partial \psi_0}{\partial x} + v_0 \frac{\partial \psi_0}{\partial z} = -\psi_1 \psi_0 \tag{9}$$

$$\frac{\partial \psi_1}{\partial x} + v_1 \frac{\partial \psi_1}{\partial z} = \psi_0 \psi_1 \tag{10}$$

where

$$\psi_0 = \frac{\beta k_0^2 \phi_0^2}{v_{gx1} \omega_0}, \quad \psi_1 = \frac{\beta k_1^2 \phi_1^2}{v_{gx0} \omega_1} \tag{11}$$

and

$$v_j = \left(\frac{v_{gz}}{v_{gx}}\right)_j . \tag{12}$$

Further, $\mathbf{v}_{gj} = [(k_{xj} k_{\perp j} \omega_j / k_j^2)\hat{x} + (k_{zj} k_{\parallel j} \omega_j / k_j^2)\hat{z}]$ can be identified with the group velocities of the two modes in the x-z plane and β the coupling coefficient is given as:

$$\beta = \frac{c^2}{B_0^2} \frac{(\mathbf{k}_1 \times \mathbf{k}_0 \cdot \hat{z})^2}{k_1^2 k_0^2} k^2 \,\mathrm{Im} \left(\frac{\chi_e (1 + \chi_i)}{1 + \chi_e + \chi_i} \right) . \tag{13}$$

Equations (9) and (10) are analogous to equations describing three-wave resonant equations with one wave heavily damped and have been analyzed previously.[7] They admit exact analytic solutions of the form

$$\psi_0 = T_\tau(\tau)/[Z(\zeta) - T(\tau)] \qquad (14)$$

$$\psi_1 = Z_\zeta(\zeta)/[Z(\zeta) - T(\tau)]\,. \qquad (15)$$

T and Z are two arbitrarily differentiable functions of $\tau = -(z - v_0x)/v$ and $\zeta = (z - v_1x)/v$, with $v = v_0 - v_1$ and can be uniquely defined by specifying boundary conditions at $x = 0$. In terms of $\psi_0(0,z)$ and $\psi_1(0,z)$ they are given as:

$$T(\tau) = -\tfrac{1}{2} - (1/v)\int_0^{-v\tau} ds\, \psi_0(0, s)\exp\{\int_0^s dr[\psi_0(0, r) + \psi_1(0, r)]\} \qquad (16)$$

$$Z(\zeta) = \tfrac{1}{2} + (1/v)\int_0^{-v\zeta} ds\, \psi_1(0, s)\exp\{\int_0^s dr[\psi_0(0, r) + \psi_1(0, r)]\}\,. \qquad (17)$$

These expression enable us to make an exact calculation of the depletion for the two-dimensional problem.

III. DEPLETION CALCULATIONS

We now need to choose appropriate forms for $\psi_0(0, z)$ and $\psi_1(0, z)$ to represent our physical problem. We let

$$\psi_0(0, z) = \begin{cases} a_0 & \text{for} \quad |z| \le 2Na \\ 0 & \quad\quad\;\; |z| > 2Na \end{cases} \qquad (18)$$

$$\psi_1(0, z) = A_1 \quad \text{for all } z \qquad (19)$$

where $2a$ is the width of the waveguide and N is the number of waveguides for a phased array source. Equations (18) and (19) model the excitation of the pump wave from a finite extent source and allow the sideband to exist as noise everywhere (it is assumed to exist as thermal fluctuations with $A_1 \ll A_0$). The sideband amplifies whenever it traverses the region of the propagation cone of the pump wave. This results in a depletion of the pump wave energy as it propagates into the plasma. We can obtain a measure of the pump power at any distance x (which in a toroidal geometry would coresond to the minor radial distance from the plasma edge) by integrating the function $\psi_0(x)$ across the resonance cone, i.e.,

$$\text{pump power} \propto F(x) = \int_{-\infty}^{\infty} dz\, \psi_0(x, z) \tag{20}$$

It is convenient, in fact, to study pump depletion in terms of the normalized quantity $P(x)$ $= [F(x)/F(0)]$, where $F(0) = 2NaA_0$ from (20). Using Eqs. (14-19) to evaluate ψ_0, ψ_1 and carrying out the integration in (20), we get

$$P(X) = \begin{cases} I_0(X)\,[1 + bI_1(X)] & \text{for} \quad bX \leq 2 \\ bI_2(X) & \text{for} \quad bX > 2 \end{cases} \tag{21}$$

where

$$I_0(X) = \frac{1 + a_1}{1 + a_1 \exp[(1 + a_1)X]} \tag{22}$$

$$I_1(X) = \int_0^X ds\, \frac{a_1 \exp[(1 + a_1)X] + \exp[(1 + a_1)s] - (1 + a_1)\exp(s + a_1 X)}{1 - \exp[(1 + a_1)s] + (1 + a_1)\exp(s + a_1 X)} \tag{23}$$

$$I_2(X) = \int_0^{1/b} ds\, \frac{1 + a_1}{1 - \exp[(1 + a_1)s] + (1 + a_1)\exp(s + a_1 X)} \tag{24}$$

$$b = v/2NaA_0 \tag{25}$$

$$a_1 = A_1/A_0 \tag{26}$$

and x has been made dimensionless by

$$X = xA_0 \,. \tag{27}$$

Equation (21) has been written in a form where the various physical factors affecting pump depletion can be clearly delineated. $I_0(X)$ represents one dimensional depletion and terms proportional to b represent two dimensional effects. The limit $b \rightarrow 0$ can arise if $v \to 0$ (i.e. the pump and sideband travel in the same direction) or if $a \to \infty$ (i.e. pump width is very large). In either case, it would correspond to the sideband continuously gaining energy from the pump, leading to a near exponential decay of the pump as given by $I_0(X)$. The depletion scale length would be of the order $1/(A_0 + A_1) \approx 1/A_0$ and as we shall later see be mainly determined by the coupling coefficient β. Finite b checks this decay in two ways: by convective loss of the sideband mode out of the pump resonance cone due to the difference in the group velocities of the two modes and by providing a limit to the interaction region on account of the finite width of the source. In Fig. 1, we have plotted $P(X)$ *vs.* X for various values of b, a fixed arbitrary A_0 and a particular (small) value of A_1/A_0 to illustrate this point.

Next, we look at the one dimensional depletion scale length $(1/A_0)$ in some detail, since it determines in a sense the maximum depletion possible in this process. Using our definitions for ψ, β etc., we can relate A_0 back to physical parameters of our problem. We get

$$A_0 = \left(\frac{cE_0}{B_0}\right)^2 \frac{k_1^2}{k_{1x}k_{1\perp}} \frac{1}{\omega_0\omega_1} \left|\frac{\mathbf{k}_1 \times \mathbf{k}_0 \cdot \hat{\mathbf{e}}_z}{k_1 k_0}\right|^2 k^2 \,\mathrm{Im}\left[\frac{\chi_e(1+\chi_i)}{1+\chi_e+\chi_i}\right] \tag{28}$$

where $E_0 = |k_0\phi_0|$ is the pump electric field. For kinetic ion quasi modes,[5] damping is primarily due to electron Landau resonance, and for $T_e \gg T_i$, we have typically $\chi_i \gg \chi_e > 1$.

Then

$$k^2 \mathrm{Im}\left(\frac{\chi_e(1+\chi_i)}{1+\chi_c+\chi_i}\right) \approx k^2 \mathrm{Im}(\chi_e) = \frac{1}{\lambda_{De}^2} \frac{\omega}{k_z v_e} \pi^{1/2} \exp(-\omega^2/k_z^2 v_e^2) \,. \tag{29}$$

Maximum damping occurs near $\omega/k_z v_e = 1/\sqrt{2}$ and A_0 can now be approximated as

$$A_0 \simeq (\pi/2)^{1/2} \frac{e^{-1/2}}{\lambda_{De}^2} \left(\frac{cE_0}{B_0}\right)^2 \frac{k_1^2}{k_{1x}k_{1\perp}} \frac{1}{\omega_0\omega_1} \left|\frac{\mathbf{k}_1 \times \mathbf{k}_0 \cdot \hat{\mathbf{e}}_z}{k_1 k_0}\right|^2 . \tag{30}$$

Similar expressions can be obtained for b and A_1 and for given plasma parameters $P(x)$ can be evaluated quantitatively.

It is also interesting to examine the variation of these quantities with density and temperature--parameters which vary across the radial cross section of a tokamak plasma. This would then provide an approximate scaling of depletion across the plasma column. We have carried out such a calculation numerically for parameters relevant to the proposed lower hybrid wave heating experiments on the ALCATOR-A machine. Typically we choose $n_{e0} = 10^{14}$ cm^{-3}, T_{e0} = 800 eV and B_0 = 50 kG. The proposed width for an array of two waveguide source is 2.62 cms, with a height of 8.2 cms and operating at a frequency of 2.45 GHz. RF power at the source is taken to be 200 kW and the electric field of the pump at any radial distance is calculated from this taking proper account of the WKB enhancement of the field. The density and temperature profiles are chosen as follows:

$$n_e = n_{e0}(1 - s^2) \tag{31}$$

$$T_e = T_{e0}(1 - s^2)^2 \tag{32}$$

where s is the dimensionless radial distance. We assume that $k_{1x} = k_{1\perp}/10$. Other quantities are derived from these parameters with the help of the constraints imposed by the parametric resonance conditions, the dispersion relations of the lower hybrid modes and maximizing the coupling coefficient.

In Fig. 2, we have plotted the assumed profiles, (31) and (32), and A_0 and b. Very near the surface A_0 rises sharply within a very narrow region so that depletion would tend to be rapid and become one dimensional. Such a tendency is primarily brought about by the fact that T_e/n_e goes to zero at the plasma edge using our assumed profiles, (31) and (32). To illustrate this fact, in Fig. 2 (b) we have used identical profiles for T_e and n_e and A_0 is found not to display such singular behaviour at the edge. Beyond the edge, in Fig. 2 (a), A_0 initially drops, two-dimensional effects take over, and the depletion is greatly reduced. Close to the lower hybrid resonance the WKB enhancement of the field causes A_0 to increase once again, and the depletion is more severe.

In obtaining quantitative estimates of depletion it is important to point out that our model applies locally at each value of x (since our calculations are for a homogeneous plasma) and we do not calculate depletion continuously across the plasma column. Further, profiles for T_e and n_e close to the edge are unknown experimentally and in addition our model equations do not apply in that region. Additional effects that may be important close to the edge are the inhomogeneity, which must be accounted for more accurately than we have, and the nonlinear saturation of the ponderomotive density fluctuations when $e\phi_p/T \sim 1$. We therefore exclude the edge region in our present discussion and examine the region from $s \simeq 0.9$ to the lower hybrid resonance layer. In Fig. 3, we have plotted $P(x)$ *vs* x assuming a fixed pump amplitude at each of the three initial values of s. In each case we assume the plasma is homogeneous with the parameters of that point. It is seen that depletion scale length decreases as we move into the plasma approaching the lower hybrid resonance layer. The amount of depletion is not very significant in these regions, for the particular parameters of ALCATOR-A.

Summarizing, our detailed analysis of the effect of parametric excitation of quasi-modes on the nonlinear pump depletion of a lower hybrid mode has shown that:

1. Two dimensional effects viz. convective losses and finite pump effects are important constraints on the pump depletion (Fig. 1).

2. Depletion is sensitive to the density and temperature profiles. In particular the parameter, A_0, becomes very large close to the edge using our assumed profiles. This indicates that a better understanding of the edge parameters may be needed before the effects of quasi-mode decay can be determined.

3. For the proposed experiment on ALCATOR-A, depletion by this parametric process is not likely to be significant.

ACKNOWLEDGEMENT

This work was supported by the National Science Foundation (Grant No. ENG 75-06242) and by the U. S. Energy Research and Development Administration (Contract No. EX-76-A-01-2295).

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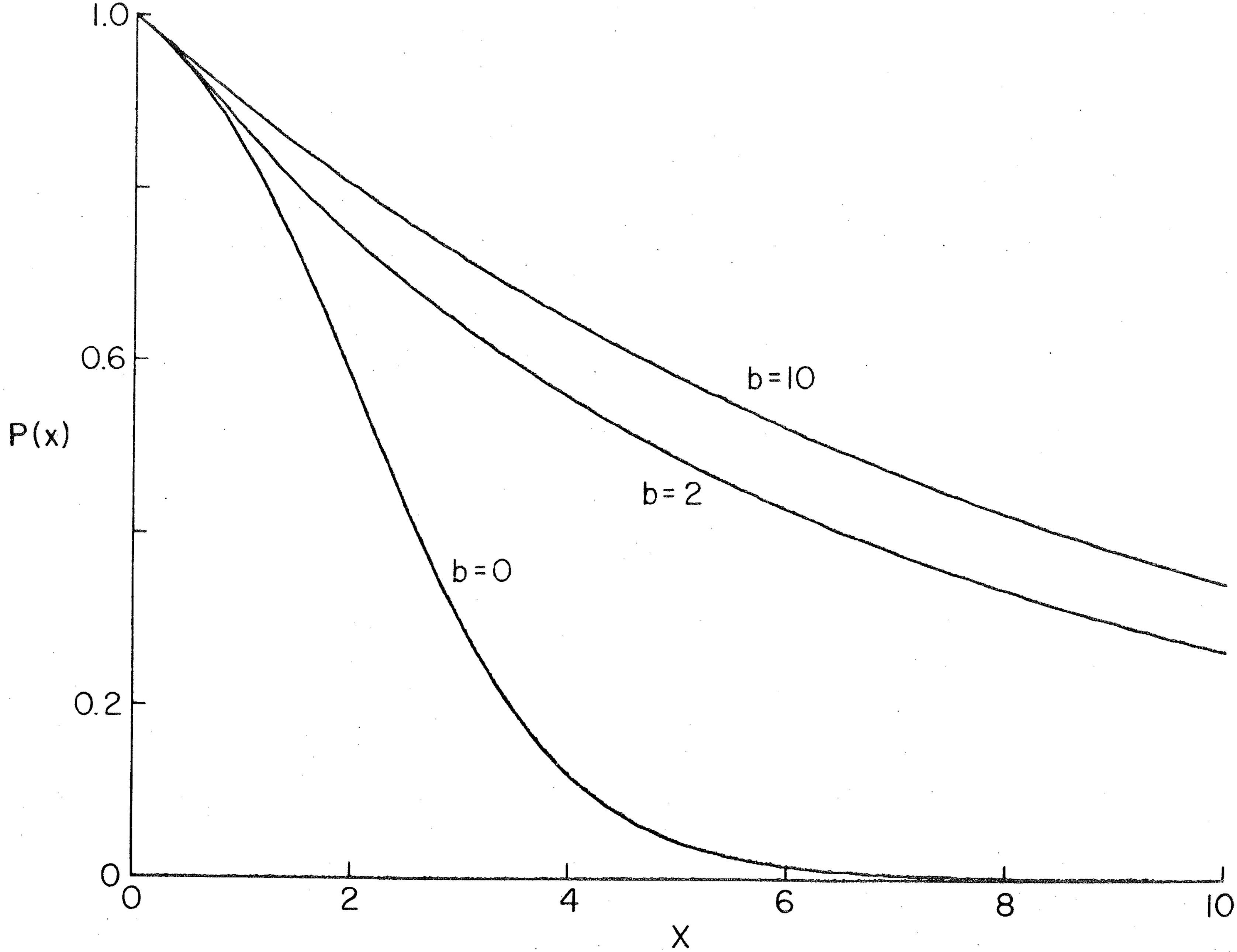
FIGURE CAPTIONS

Fig. 1 Plots of P(X) vs. X for $b = 0$, $b = 2$ and $b = 10$. The value of A_1/A_0 is fixed at 0.1.

Fig. 2 Profiles of n_e/n_{e0}, T_e/T_{e0}, A_0 and b for (a) $n_e/n_{e0} = (1 - s^2)$, $T_e/T_{e0} = (1 - s^2)^2$ and (b) $n_e/n_{e0} = T_e/T_{e0} = (1 - s^2)$. A_0 and b have been normalised to their maximum values.

Fig. 3 Plots of P(x) vs. x originating at $s = 0.75$, $s = 0.5$ and $s = 0.3$. The lower hybrid resonance layer is close to $s = 0.2$.



1.0
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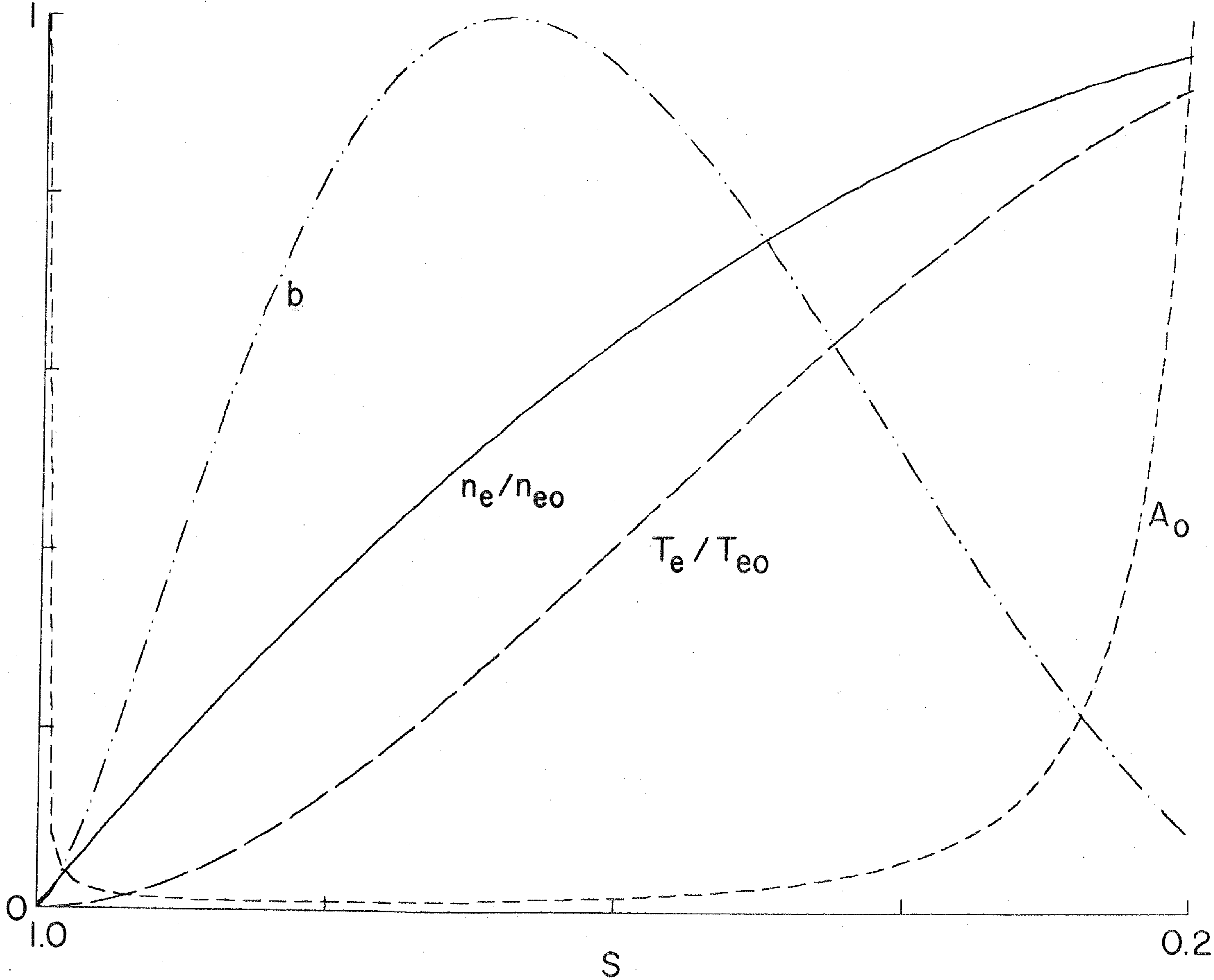
0.2
0
P(x)
b=10
b=2
b=0
0
2
4
6
8
10
X



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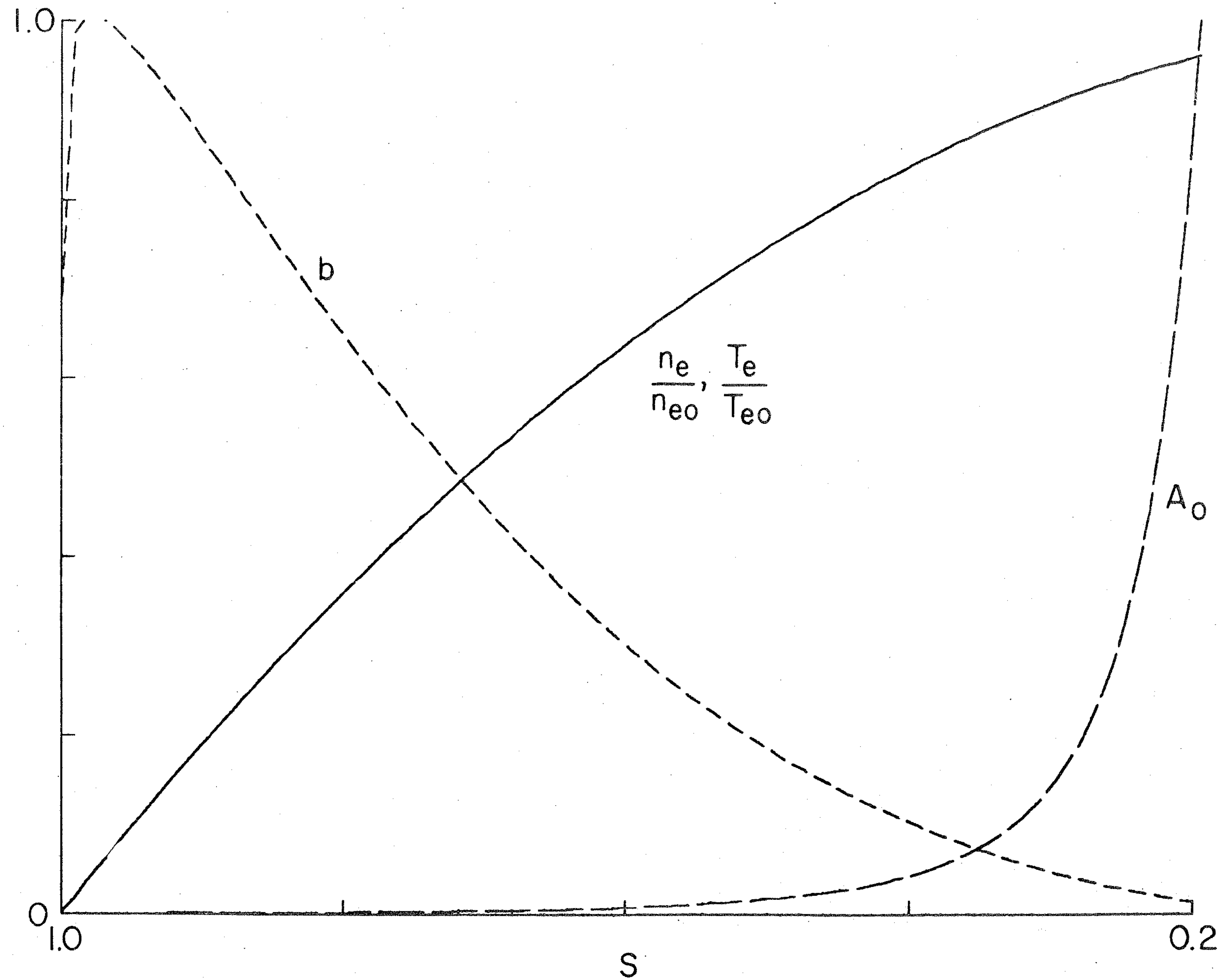
0.2
S
b
n_e/n_{eo}
T_e/T_{eo}
A_o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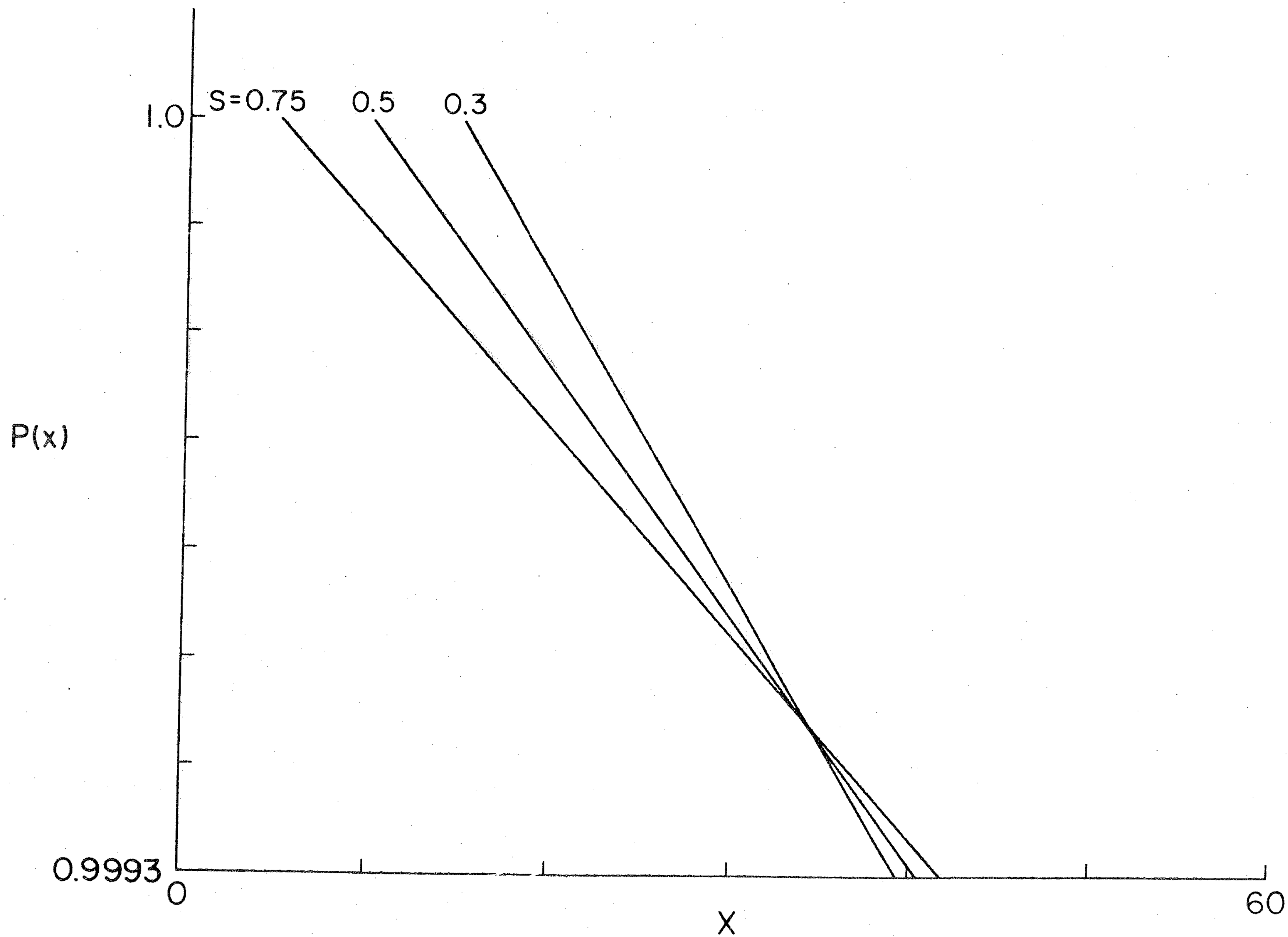
1.0
0.2
S
b
$\frac{n_e}{n_{eo}}, \frac{T_e}{T_{eo}}$
A_0



S=0.75
0.5
0.3
1.0
P(x)
0.9993
0
60
X