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TEMPORAL EVOLUTION OF NONLINEAR LOWER HYBRID WAVES*

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Abstract

The nonlinear evolution of a single lower hybrid waves in two dimensions and time is considered. If the potential is taken to have the form $\phi(x,z,t)\exp(-i\omega t)$, the equation describing the fields is $iv_\tau - \int v_\xi \, d\zeta + v_{\zeta\zeta} + |v|^2 v = 0$ where v is proportional to the electric field, τ to time, and ξ and ζ are distances along and across the lower hybrid ray. The properties of this equation are investigated numerically. When the amplitude of the injected lower hybrid waves is sufficiently large, the following phenomena are observed: the fields do not reach a steady state even though steady-state boundary conditions are imposed; the density modulations are sufficient to cause an appreciable fraction of the incident power to be reflected; the average wavenumber of the transmitted wave is larger than that of the incident wave.

Introduction

The injection of rf power near the lower hybrid frequency is an attractive method for the auxiliary heating of tokamak plasmas.[1] Because of the high powers required and because the propagation of lower hybrid waves is principally along well-defined resonance cones, there has been considerable interest in nonlinear effects on the propagation of lower hybrid waves. This problem was first addressed by Morales and Lee[2] who studied the two-dimensional electrostatic propagation of one of the two lower hybrid rays in a homogeneous plasma. In formulating this problem, they assumed that the fields

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in the plasma had reached a steady state, i.e. that the potential is given by $\mathrm{Re}[\phi(x,z)\exp(-i\omega t)]$ (x and z are coordinates perpendicular and parallel to the ambient magnetic field B). The electric field is then found to obey the complex modified Korteweg-deVries equation.[2,3,4] When the correct boundary conditions are applied, it is found that this equation is ill-posed.[4] The problem arises because the direction of power flow which determines how to impose the boundary conditions is defined only with reference to a problem in which a temporal evolution of the wave packet is allowed. In assuming a steady state for the electric field amplitude, the equation no longer has built into it the crucial ingrediant which determines how the boundary conditions should be imposed.

We correct this deficiency by allowing the potential to have a slow temporal variation, i.e, $\mathrm{Re}[\phi(x,z,t)\exp(-i\omega t)]$ where the t dependence of ϕ is taken to be much slower than ω.

In the remainder of this paper, we will write down the basic equations governing the propagation of the waves, give the results of numerically integrating this equation, and discuss the implications of our results to the lower hybrid heating of tokamaks. A fuller account of this work is given in Ref. 5.

Basic Equations

In order to derive the equations for the fields we closely followed the formulation of Morales and Lee.[1] The fields are taken to be electrostatic and the plasma is assumed to be homogeneous and to be immersed in a uniform magnetic field. In the linear non-dispersive limit, Poisson's equation for the fields in the (x,z) plane reduces to the wave equation (where one of the spatial coordinates plays the role of time). The general solution is the superposition of a left- and a right-going ray (the two resonance cones). We assume that only the right-going ray is present and we include (to lowest order) the effects of a slow time evolution of the waves, of thermal dispersion, and of modification of the ray direction due to the ponderomotive density fluctuations. To leading order, the lower hybrid waves obey

$$iv_\tau - \int_{-\infty}^{\zeta} v_\xi \, d\zeta + v_{\zeta\zeta} + |v|^2 v = 0 \,. \tag{1}$$

Here Greek letters denote differentiation and

$$v = \left(\frac{\varepsilon_0}{4n_0T_e}\right)^{1/2} E \,, \quad \tau = \frac{\omega_0 t}{2} \,, \quad \zeta = \frac{gz - x}{\sqrt{3}\,\lambda_{De}} \,, \quad \xi = \frac{x}{2\sqrt{3}\,\lambda_{De}} \,. \tag{2}$$

E is the electric field measured in the direction perpendicular to the lower hybrid ray and g is ω/ω_{pe}. As expected, Eq. (1) reduces to the complex modified Korteweg-deVries equation in the limit $\partial/\partial\tau = 0$.

Before solving Eq. (1), we must determine the correct boundary conditions to apply. To do this, we Fourier transform Eq. (1) in the ζ direction to obtain

$$V_\tau + cV_\xi + i\Omega V + iN(V) = 0 \tag{3}$$

where $c = 1/\kappa$, $\Omega = \kappa^2$, N is the Fourier transform of the nonlinear term, and κ is the wavenumber in the ζ direction. This equation is hyperbolic as far as the (τ,ξ) coordinates are concerned. However the direction of the characteristic velocity c depends on κ. This means that we must specify boundary conditions at each end of a strip in ξ. If the domain of the problem is

$$\xi_0 < \xi < \xi_1 \ , \quad -\infty < \zeta < \infty \quad , \quad \tau_0 < \tau < \infty \ , \tag{4}$$

then the initial and boundary conditions required are

$$V(\tau = \tau_0) \ , \quad V(\xi_0, \kappa > 0) \ , \quad V(\xi_1, \kappa < 0) \ . \tag{5}$$

Recognizing that c is the group velocity for the lower hybrid waves, we can see that these boundary conditions are equivalent to specifying the waves incident on the strip. Furthermore, in order to determine these boundary conditions, we had to consider the problem with the temporal variation included. If we now attempt to apply these boundary conditions to the equation obtained by setting $\partial/\partial\tau = 0$, we need not end up with a well-posed problem.

Numerical Solution

We investigate the solution to Eq. (1) by numerically solving Eq. (3) with boundary and initial conditions

$$V(\tau = 0) = 0 \ , \quad V(\xi_1, \kappa < 0) = 0 \ ,$$

$$V(\xi = 0, \kappa > 0) = v_0\pi \ u(\kappa - \kappa_0)(\kappa - \kappa_0) \exp[-\tfrac{1}{4} (\kappa - \kappa_0)^2] \ ,$$

where u is the unit step function, and v_0 and $\kappa_0 \geq 0$ are real constants. Without loss of generality, we have taken $\tau_0 = \xi_0 = 0$. The solution depends on three parameters: v_0, κ_0, and $\Delta\xi \equiv \xi_1 - \xi_0 = \xi_1$.

In the limit of small v_0, the various κ modes propagate across the system at velocity c. Once all the modes have traversed the system, the fields reach a steady state in which $|V|$ is a function of κ only.

As the amplitude of the boundary condition is increased, more complicated behavior sets in. Figure 1 shows the solution with $v_0 = 4$, $\kappa_0 = 0$, and $\Delta\xi = 0.1$. The electric field v appears never to reach a steady state. Instead, it oscillates in an aperiodic fashion. From Fig. 1, we can see what is happening during these oscillations. Various κ components of the forward wave nonlinearly interact to produce a reflected wave, with κ negative [Fig. 1(b)]. This interaction causes a severe depletion of the low κ components of the forward wave and the transfer of this energy into higher κ components of

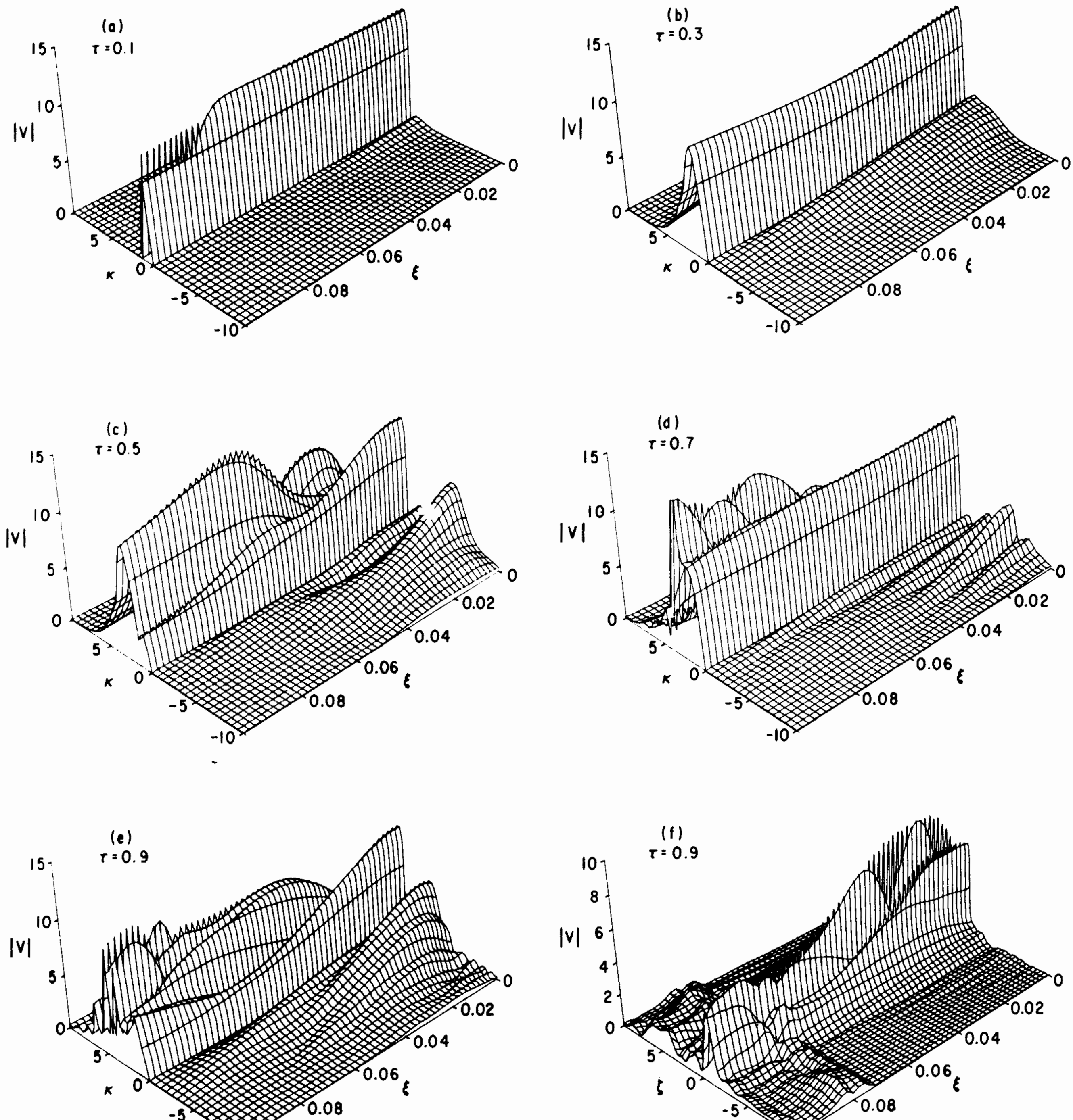

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Fig. 1. The solution to Eq. (1) for $v_0 = 4$, $\kappa_0 = 0$, and $\Delta\xi = 0.1$. The solution is shown in κ space for (a) $\tau = 0.1$, (b) $\tau = 0.3$, (c) $\tau = 0.5$, (d) $\tau = 0.7$, (e) $\tau = 0.9$, and in ζ space for (a) $\tau = 0.9$.

the forward wave and into the reflected wave [Fig. 1(c)]. After the interaction has nearly gone to completion, the nonlinearly excited components of the field transit out of the system and the fields relax to a state in which there is little variation of $|V|$ with ξ [Fig. 1(d)]. The nonlinear interaction then begins again and the cycle approximately repeats itself [Fig. 1(e)]. In ζ space, this interaction is manifested by a narrowing and peaking of the electric field amplitude v. A typical field pattern is shown in Fig. 1(f).

In this case, the average amount of the incident power reflected is about 20%. The average wavenumber of the transmitted waves is about twice that of the incident waves. In Fig. 2, we show the temporal spectrum of the transmitted and reflected waves. This is obtained by taking a time record $1 < \tau < 5$ of the transmitted ($\xi = \xi_1$ and $\kappa > 0$) and reflected ($\xi = 0$ and $\kappa < 0$) waves and transforming it into σ space where σ is the shift from the incident frequency ω. The broad spectrum in σ is symptomatic of the aperiodic turbulent nature of the waves. There is an interesting correlation in the spectrum: the positive frequency (σ) components of both the transmitted and reflected waves tend to have larger values of $|\kappa|$ than the negative frequency components.

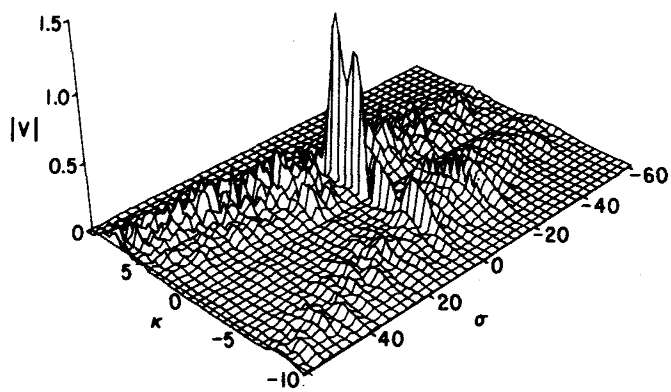

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Fig. 2. The output spectrum for the case shown in Fig. 1.

Thus, three phenomena of interest to lower hybrid heating experiments are observed: (1) the reflection coefficient is appreciable; (2) the solution reaches a turbulent state; (3) the mean wavenumber of the waves transmitted into the plasma is increased. These phenomena pretty much occur together. They disappear if v_0 or $\Delta\xi$ is decreased or if κ_0 is increased. If we compare these results with those of Ref. 4 in which the solution of the steady-state problem was attempted, we see that the conditions for the occurrence of these phenomena closely agree with the conditions under which the steady-state problem had no solution and reflection was large. Quoting these conditions from Ref. 4, we have

$$E_{z0}\,\delta z/T_e \gtrsim 4\sqrt{3}\pi \;, \qquad \Delta x \gtrsim g\,\Delta z/\beta \;, \tag{6}$$

where E_{z0} and E_{x0} are the electric field amplitudes parallel and perpendicular to the magnetic field, Δz is the width of the waveguide array, δz is the width of a single waveguide (assuming a 0, π, 0, π, ... phasing), Δx is the width of the nonlinear region of the plasma in the x direction, and β is $\frac{1}{4}\varepsilon_0 E_{x0}^2/n_0 T_e$, the ratio of the electric field energy to the plasma kinetic energy.

Discussion

We have examined the nonlinear evolution of a lower hybrid wave in two dimensions and time. Under the conditions given in Eq. (6), the nonlinearity can cause appreciable reflection, turbulent variation in the fields, and an increase in the wavenumbers of the transmitted waves. In such circumstances, the assumption of a steady state and the analyses in Refs. 2-4 based on this assumption are wrong.

In Ref. 4, it was found that the nonlinearity could be important close the edge of a tokamak plasma. Unfortunately, it is not possible to obtain quantitative estimates of these effects in the edge region with the theory as outlined above because many other physical processes are likely to be involved (e.g., coupling to the other ray, electromagnetic effects, density and temperature gradients, saturation of the nonlinearity, ion inertia in the low frequency equations, coupling to low-frequency drift waves).

Thus, Eq. (1) should be regarded as a very simplified model equation describing the propagation of lower hybrid waves near the edge of a tokamak plasma. Nevertheless, some of the phenomena predicted by this equation have been observed in the lower hybrid heating experiment on Alcator-A.[6,7] There, a broadening of the frequency spectrum was observed. This is consistent with the turbulent spectra seen in the solution of Eq. (1). Furthermore the spectrum becomes asymmetric as the wave propagates into the plasma, the peak being shifted down from the frequency of the injected waves. This may be a result of the phenomenon seen in Fig. 2 where we found that the components of the waves which are down-shifted have a smaller wavenumber than those which are up-shifted. If those waves with higher wavenumbers are preferentially damped as the wave travels into the plasma (e.g., by electron Landau damping), we would expect to see a net downwards shift in the spectrum of the waves.

In addition, it was found on the Alcator-A experiment that the wavenumbers of the waves transmitted into the plasma was independent of the phasing of the waveguides and that these wavenumbers were higher than those predicted from linear theory even for the waveguides being out of phase. Again this is in qualitative agreement with the numerical results. In the case shown in the figures, the average wavenumber of the wave transmitted into the plasma was about twice that of the incident waves. As κ_0 is increased (i.e., as the relative phasing between waveguides is increased), the amount by which the wavenumber is increased is reduced.

It is not clear whether the nonlinear reflection predicted would be observed as an increase in the reflected power measured in the waveguides. It may be that this power is reflected again on the cutoff at $\omega = \omega_{pe}$ very close to the plasma edge. This would convert the power into the other lower hybrid ray.

It appears that the physics included in Eq. (1) may be responsible for some of the results of the Alcator-A experiment. In order to be able to say definitively whether or not the experimental observations are a result of the processes included in this equation, the theory would have to be refined to a point where quantitative comparisons are possible.

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