Two-dimensional depletion of a lower hybrid pump by quasi-mode excitations

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The depletion of a finite extent lower hybrid pump wave due to the parametric excitation of quasi-modes is calculated for parameters relevant to tokamak plasmas.

In this note we give results of a detailed investigation of the effect of parametric excitation of quasi-modes on the depletion of a lower hybrid pump excited from a finite extent source. Basically, we consider the coupling of the electrostatic pump wave potential $\phi_0(x) \times\exp(-i\omega_0 t)$ with another lower hybrid wave potential $\phi_1(x)\exp(-i\omega_1 t)$ such that the beat wave with $\omega=\omega_0-\omega_1$ and $\mathbf{k}=\mathbf{k}_0-\mathbf{k}_1$ has a phase velocity close to the electron thermal velocity and consequently is heavily damped. The electron density fluctuations corresponding to this low frequency beat mode provide the necessary nonlinear coupling between ϕ_0 and ϕ_1 and the nonlinear coupled equations can be written in the form[1]

$$\frac{\partial\psi_0}{\partial x}+v_0\frac{\partial\psi_0}{\partial z}=-\psi_1\psi_0\ ;\ \frac{\partial\psi_1}{\partial x}+v_1\frac{\partial\psi_1}{\partial z}=\psi_0\psi_1\ , \tag{1}$$

where $\psi_0=(\beta k_0^2\phi_0^2)/(v_{gx1}\omega_0)$, $\psi_1=(\beta k_1^2\phi_1^2)/(v_{gx0}\omega_1)$, and $v_j=(v_{gz}/v_{gx})_j$ are ratios of the group velocities of the two modes in the x-z plane. The coupling coefficient β is

$$\beta=\frac{1}{B_0^2}\frac{(\mathbf{k}_1\times\mathbf{k}_2\cdot\hat{z})^2}{k_1^2k_0^2}k^2\,\mathrm{Im}\left(\frac{\chi_e(1+\chi_i)}{1+\chi_e+\chi_i}\right), \tag{2}$$

where χ_e, χ_i are the electron and ion susceptibilities of the quasi-mode and B_0 is the ambient magnetic field. Equations (1) are analogous to equations describing three-wave resonant equations with one wave heavily damped and have recently been analyzed.[2] They admit of exact analytic solutions of the form

$$\psi_0=T_\tau(\tau)[Z(\xi)-T(\tau)]^{-1}\ ,\quad \psi_1=Z_\xi(\xi)[Z(\xi)-T(\tau)]^{-1}\ . \tag{3}$$

T and Z are two arbitrarily differentiable functions of $\tau=-(z-v_0x)/v$ and $\xi=(z-v_1x)/v$, with $v=v_0-v_1$ and can be uniquely defined by specifying boundary conditions at $x=0$. In terms of $\psi_0(0,z)$ and $\psi_1(0,z)$ they are

$$T(\tau)=-\tfrac{1}{2}-(1/v)\int_0^{-v\tau}ds\,\psi_0(0,s)J(s) \tag{4}$$

$$Z(\xi)=\tfrac{1}{2}+(1/v)\int_0^{-v\xi}ds\,\psi_1(0,s)J(s) \tag{5}$$

with

$$J(s)=\exp\left\{\int_0^s dr[\psi_0(0,r)+\psi_1(0,r)]\right\}\ .$$

We let

$$\psi_0(0,z)=A_0\ ,\ \text{for}\ |z|\le Na\ ,\qquad 0\ \text{for}\ |z|>Na\ , \tag{6}$$

$$\psi_1(0,z)=A_1,\ \text{for all}\ z, \tag{7}$$

where $2a$ is the effective width of the waveguide and N is the number of waveguides for a phased array source. Equations (6) and (7) model the excitation of the pump wave from a finite extent source and allow the sideband to exist as noise everywhere (it is assumed to exist as thermal fluctuations with $A_1\ll A_0$). We study pump depletion in terms of the normalized quantity $P(x)=\int_{-\infty}^{\infty}dz\,\psi_0(x,z)/2NaA_0$. Using Eqs. (3)–(7), we obtain

$$P(X)=I_0(X)[1+bI_1(X)]\ ,\ \text{for}\ bX\le 2,\quad bI_2(X),\ \text{for}\ bX>2, \tag{8}$$

where

$$I_0(X)=\frac{1+a_1}{1+a_1\exp[(1+a_1)X]}\ , \tag{9}$$

$$I_1(X)=\int_0^X ds \times\frac{a_1\exp[(1+a_1)X]+\exp[(1+a_1)s]-(1+a_1)\exp(s+a_1X)}{1-\exp[(1+a_1)s]+(1+a_1)\exp(s+a_1X)}\ , \tag{10}$$

$$I_2(X)=\int_0^{1/b}ds\,\frac{1+a_1}{1-\exp[(1+a_1)s]+(1+a_1)\exp(s+a_1X)}\ , \tag{11}$$

with $b=v/2NaA_0$, $a_1=A_1/A_0$, and x has been made dimensionless by $X=xA_0$. In (8) $I_0(X)$ represents one-dimensional depletion and terms proportional to b represent two dimensional effects. The limit $b\to 0$ can arise if $v\to 0$ (i.e., the pump and sideband travel in the same direction) or if $a\to\infty$ (i.e., pump width is very large). In either case, it would correspond to the sideband continuously gaining energy from the pump, lead-

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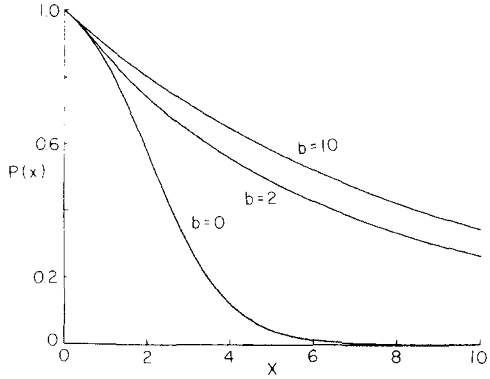

FIG. 1. Plots of $P(x)$ vs X for $b=0$, $b=2$, and $b=10$. The value of A_1/A_0 is fixed at 0.1.

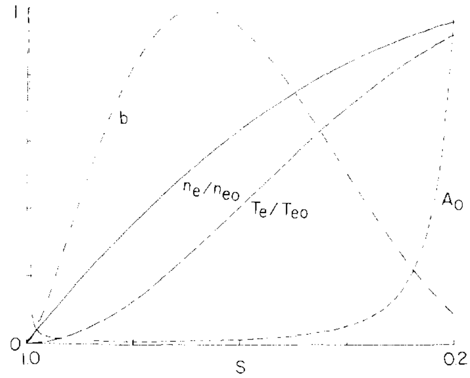

FIG. 2. Profiles of n_e/n_{e0}, T_e/T_{e0}, A_0, and b for $n_e/n_{e0} = (1-s^2)$, $T_e/T_{e0} = (1-s^2)^2$. A_0 and b are normalized to their respective maximum values.

ing to a near exponential decay of the pump as given by $I_0(X)$. The depletion scale length would be of the order $1/(A_0+A_1) \approx 1/A_0$ and is mainly determined by the coupling coefficient β. A nonzero v and finite a, and hence nonzero b, allow the sideband to propagate out of the pump region; this is a convective loss to the interaction and reduces pump depletion. In Fig. 1 we have plotted $P(X)$ vs X for various values of b and a particular (small) value of A_1/A_0 to illustrate this point.

Next, we maximize the one dimensional depletion scale length $(1/A_0)$ since in a sense it determines the maximum depletion possible in this process. For kinetic ion quasi-modes,[3] damping is primarily due to electron Landau resonance, and for $T_e \gg T_i$, we have typically $\chi_i \gg \chi_e > 1$, so that in the last term in (2) $\mathrm{Im}\,\chi_e$ dominates. (For $T_e \approx T_i$ the same mode is still important but for shorter low frequency wavelengths[4]). The maximum damping occurs near $\omega/k_z v_e = 1/\sqrt{2}$ and using our definitions for ψ, β, etc., we can express A_0 as

$$A_0 \simeq \left(\frac{\pi}{2}\right)^{1/2} \frac{0.607}{\lambda_{De}^2} \left(\frac{E_0}{B_0}\right)^2 \frac{k_1^2}{k_{1x}k_{1\perp}} \frac{1}{\omega_0\omega_1} \left|\frac{k_1 \quad k_0 \cdot \hat{e}_z}{k_1 k_0}\right|^2 . \qquad (12)$$

where $E_0 = |k_0\phi_0|$ is the pump electric field and $K_{1\perp}$ is the perpendicular component of the cold dielectric function at the sideband frequency. Similar expressions can be obtained for b and A_1 and for given plasma parameters $P(x)$ can be evaluated quantitatively.

Finally, we examine the variation of these quantities with density and temperature, to get an approximate scaling of depletion across the plasma column. We have carried out such a calculation numerically for parameters relevant to tokamak plasmas. We choose $n_{e0} = 10^{14}$ cm^{-3}, $T_{e0} = 800$ eV, and $B_0 = 50$ kG which are typical of the ALCATOR-A plasma,[5] and let $k_{0z} = 2.4$, $\omega_0 = 1.54 \times 10^{10}$ to characterize the spectrum of a two-waveguide excitation near the maximum lower hybrid frequency. The electric field at the edge is taken to be about 4 kV/cm and the appropriate WKB enhanced value is used in the interior of the plasma. The density and temperature profiles are chosen as follows: $n_e = n_{e0}(1-s^2)$, $T_e = T_{e0}(1-s^2)^2$, where s is the dimensionless radial distance.

In Fig. 2, we have plotted the assumed profiles, and A_0 and b. Very near the surface A_0 rises sharply within a very narrow region so that depletion would tend to be rapid and become one dimensional. Such a tendency is primarily brought about by the fact that T_e/n_e goes to zero at the plasma edge using our assumed profiles. (For example, when we use identical profiles for T_e and n_e, A_0 is found not to display such singular behavior at the edge.) Beyond the edge, in Fig. 2, A_0 initially drops, two-dimensional effects take over, and the depletion is greatly reduced. Close to the lower hybrid resonance the WKB enhancement of the field causes A_0 to increase once again, and the depletion is more severe.

In obtaining quantitative estimates of depletion it is important to point out that our model applies locally at each value of x (since our calculations are for a homogeneous plasma), and we do not calculate depletion continuously across the plasma column. Further, profiles for T_e and n_e close to the edge are unknown experimentally and in addition our model electrostatic equations do not apply in that region. We therefore exclude the edge region in our present discussion and examine the region from $s \approx 0.9$ to the lower hybrid resonance layer. At various points in the profile, we calculate $P(X)$ over a distance X such that in terms of the unnormalized quantity x the distance is larger than the minor radius of the plasma (e.g., 10 cm for this example.) We find that the depletion scale length decreases as we move into the plasma approaching the lower hybrid resonance layer, and the amount of depletion is very small ($\ll 1\%$) for the particular parameters of current interest.

Summarizing, our detailed analysis of the effect of parametric excitation of quasi-modes on the nonlinear pump depletion of a lower hybrid mode has shown that two dimensional effects, viz., convective losses and finite pump effects importantly reduce pump depletion, and for current experiments on tokamak plasmas, depletion by this parametric process is not likely to be significant.

The results of this note (for details see Ref. 1) differ from those in Ref. 6; however, in a more recent publication[7] of these authors, although numerical calculations for the two-dimensional case are not given, their main results and conclusions agree with ours.

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[1]A. Sen, C. F. F. Karney, and A. Bers, Research Laboratory of Electronics, Massachusetts Institute of Technology, Plasma Research Report PRR 77/19 (1977).

[2]F. Y. F. Chu and C. F. F. Karney, Phys. Fluids **20**, 1728 (1977).

[3]M. Porkolab, Phys. Fluids **17**, 1432 (1974).

[4]M. Porkolab, Phys. Fluids **20**, 2058 (1977).

[5]E. Apgar, B. Coppi, A. Gondhalekar, H. Helava, D. Komm, F. Martin, B. Montgomery, D. Pappas, R. Parker, and D. Overskei, in *Plasma Physics and Controlled Nuclear Fusion Research* (International Atomic Energy Agency, Vienna, Austria, 1977), Vol. I, p. 247.

[6]L. Chen and R. L. Berger, Bull. Am. Phys. Soc. **21**, 1156 (1976).

[7]L. Chen and R. L. Berger, Nucl. Fusion **17**, 779 (1977).

 0031-9171/78/2105-0863$00.90