

COMMENTS

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Comment on "Resonant parametric excitations driven by lower-hybrid fields"

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(Received 9 February 1981; accepted 14 May 1981)

A recent article¹ contains serious errors regarding the existence of temporally growing modes for parametric instabilities in homogeneous plasma are approximately described by²

$$\begin{aligned} \left(\frac{\partial}{\partial t} + v_1 \frac{\partial}{\partial x} + \gamma_1\right) a_1 &= \gamma_0(x) \exp\left(\frac{i\kappa x^2}{2}\right) a_2, \\ \left(\frac{\partial}{\partial t} + v_2 \frac{\partial}{\partial x} + \gamma_2\right) a_2 &= \gamma_0(x) \exp\left(\frac{-i\kappa x^2}{2}\right) a_1, \end{aligned} \quad (1)$$

where a_1 and a_2 are complex and the other quantities are real. We take $\gamma_{1,2} > 0, \kappa \neq 0$, and consider only the case $v_2 < 0 < v_1$. Rosenbluth² found that when $\gamma_0(x)$ is a constant and Eq. (1) is solved in an infinite domain in x , no temporally growing normal modes [where $a_{1,2} \sim \exp(pt)$ with $\text{Re}(p) > 0$] are possible. Dubois *et al.*³ treated the case where

$$\gamma_0(x) = \begin{cases} \gamma_0, & \text{for } |x| < \frac{L}{2}, \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

with γ_0 being a constant. The boundary conditions appropriate for normal mode solutions are

$$a_1(-L/2) = 0, \quad a_2(L/2) = 0. \quad (3)$$

They found that temporally growing solutions are possible even for arbitrarily large L . As an example, if $v_1 = -v_2 = v$, $\gamma_0 > 0$, $\gamma_1 = \gamma_2 = \gamma$, and $\kappa = \gamma_0^2/(2v^2)$, it is found^{3,4} that for large L , $(p + \gamma)/\gamma_0 = 0.6439 \pm i[L\gamma_0/(8v) - 0.5497]$, so that $\text{Re}(p) > 0$ for $\gamma/\gamma_0 < 0.6439$. This analysis has been refined^{4,5} and the apparent contradiction with Rosenbluth's result has been resolved.

Recently, Villalón¹ has published an analysis of the

system treated by Dubois *et al.*³ It is concluded that the temporally growing modes always disappear for sufficiently large L (in contradiction to Refs. 3–5). It is further stated that Dubois *et al.* are in error because the boundary conditions (3) are inapplicable. This is wrong. Results obtained with other boundary conditions do not apply to normal modes.

It may be worth reiterating the resolution of the results of Rosenbluth and Dubois *et al.* Two observations are necessary. First, the modes by Dubois *et al.* are localized near $x = \pm L/2$ so that the time taken to establish the normal modes is about $L/[2 \max(v_1, -v_2)]$ which is the time it takes for a pulse to travel from $x = 0$ to one of the edges of the system. Second, two limits are involved: in defining normal modes we must let $t \rightarrow \infty$; in order to treat the infinite system we must let $L \rightarrow \infty$. Rosenbluth takes $L \rightarrow \infty$ before $t \rightarrow \infty$ so that normal modes can never appear. Dubois *et al.* take $t \rightarrow \infty$ first so that temporally growing normal modes can exist no matter how large L is.

This work was supported by United States Department of Energy Contract No. DE-AC02-76-CH03073.

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²M. N. Rosenbluth, *Phys. Rev. Lett.* **29**, 565 (1972).

³D. F. Dubois, D. W. Forslund, and E. A. Williams, *Phys. Rev. Lett.* **33**, 1013 (1974).

⁴F. W. Chambers and A. Bers, *Phys. Fluids* **20**, 466 (1977).

⁵V. Fuchs and G. Beaudry, *Phys. Fluids* **21**, 280 (1978).