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Comparison of the theory and the practice of lower-hybrid current drive

C. F. F. Karney, N. J. Fisch, and F. C. Jobes
Plasma Physics Laboratory, Princeton University, P.O. Box 451, Princeton, New Jersey 08544
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The theory of rf-driven plasma currents is applied to the lower-hybrid experiments on the Princeton Large Torus tokamak. Particular emphasis is placed on those experiments in which the plasma current was varying. The comparison between theory and experiment is made with respect to the efficiency with which rf energy was converted to poloidal magnetic field energy. Good agreement is found irrespective of whether the current was increasing, constant, or decreasing.

Lower-hybrid waves have been used to increase the toroidal current in tokamaks.[1-7] The efficiency of this process is expressed as the ratio of the rate of change of the poloidal field energy $\dot{W}$ $[W=(\frac{1}{2})LI^2]$ to the rf-power injected $P_{\rm rf}$. In an experiment at the Princeton Large Torus, Princeton Plasma Physics Laboratory, Princeton University (PLT) an efficiency of about 25% has been reported.[6,7]

Recently, a theory of this process (rf-current ramp-up) has been proposed.[8] This theory generalizes the more restricted theory of steady-state[9,10] and nearly steady-state[11] current drive and, in this Rapid Communication, we analyze the data from the PLT in light of this generalization. Details of the experiment appear elsewhere.[6,7,12]

The theory identifies two dimensionless parameters and derives a relationship between them. One parameter is the ratio $P_{\rm el}/P_{\rm in}$, where $P_{\rm in}$ is the rf power absorbed by the hot current-carrying electrons (presumably some fraction η of $P_{\rm rf}$), and $P_{\rm el}$ is the power flowing from these electrons into the poloidal field. The second parameter is $u=v_{\rm ph}/v_R$, where $v_{\rm ph}$ is the parallel phase velocity of the lower-hybrid waves in the plasma and v_R is the runaway velocity defined by $v_R^2=|nq^3\ln\Lambda/4\pi\epsilon_0^2Em|$. We adopt the sign convention that $v_R>0$ in the "normal" ramp-up cases, i.e., when there is an electric field opposing the rf current, and $v_R<0$ when the electric field is in the same direction as the rf current. This parameter reflects the relative importance of the electric field and collisions in slowing down the hot electrons. When collisions dominate the slowing down ($u \ll 1$), we expect poor ramp-up efficiency, whereas when the electric field dominates ($u \gg 1$), we expect an efficiency close to unity.

The experimental data, on the other hand, are often plotted as efficiency versus $P_{\rm rf}$. In order to compare theory with experiment, we must relate the experimental observables to the theoretical parameters. Assuming that $P_{\rm in}=\eta P_{\rm rf}$, we may write

$$\eta\frac{P_{\rm el}}{P_{\rm in}}=\frac{P_{\rm el}}{P_{\rm rf}}=\frac{\dot{W}-P_{\rm ext}+V^2/R}{P_{\rm rf}} . \quad (1)$$

The expression for $P_{\rm el}$ is derived by balancing contributions to $\dot{W}$. These are $P_{\rm el}$, $P_{\rm ext}$ (the power coupled from the vertical and Ohmic coils), and V^2/R (the Ohmic dissipation). The second theoretical parameter u, may be determined using $v_{\rm ph}=c/n_\parallel\beta$, where $n_\parallel$ is the parallel index of refraction of the peak of the spectrum of the launched waves and β is the factor by which the peak is upshifted when the wave propagates into the plasma. Because the function defined in Eq. (2) is smooth, it is possible to characterize the spectrum by a single $n_\parallel$ component. The runaway velocity v_R is given directly in terms of experimental observables.

Figure 1 shows the experimental data from the PLT plotted in terms of $P_{\rm el}/P_{\rm rf}$ and u. This includes the results of over 250 shots with various values of density, waveguide phasing, and rf power. These shots constitute all the ramp-up experiments on the PLT during a period of two months, including those cases where the current was steady or decaying, as long as the duration of the rf pulse exceeded 200 ms. In view of the wide parameter space covered by these shots, the data show remarkably little scatter, which provides strong evidence that the crucial dimensionless parameters have been identified.

In Ref. 8, a function $G=G(u,Z)$ is defined, where Z is the effective ion-charge state, and the relation

$$P_{\rm el}/P_{\rm in}=(\partial G/\partial u)/u \quad (2)$$

is derived by ignoring rf-generated runaways, which are not expected to be confined long in the PLT experiment. Experimental estimates, based on the total level of hard-x-ray

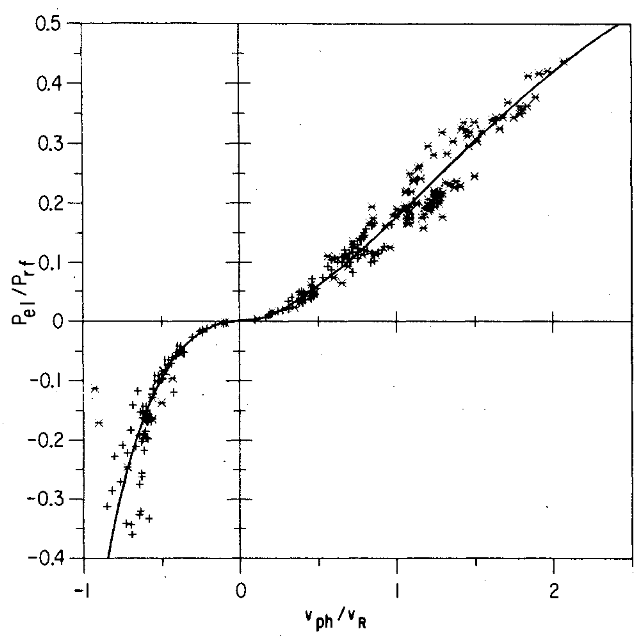

FIG. 1. $P_{\rm el}/P_{\rm rf}$ vs u for 250 PLT shots. The rf power $P_{\rm rf}$ varied from 0 to 300 kW, the density n from 1.5×10^{12} to 6.0×10^{12} cm^{-3}, the plasma current I from 150 to 400 kA. Three waveguide phasings were used 60°(*), 90°(+), and 135°(#).

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emissions, suggest $Z \simeq 5$. Hence, we take $G = G(u, 5)$ in comparing theory with experiment in Fig. 1. Note however, that G is not very sensitive to its Z dependence.

Two adjustable parameters must be chosen in order to compare theory and experiment. They are the absorption factor η and the $n_\parallel$ upshift factor β. These parameters act as scale factors on the two axes. The best fit, displayed in Fig. 1, is given when $\beta = 1.4$ and $\eta = 0.75$. Therefore in the best case, corresponding to $P_{el}/P_{rf} = 43\%$, the theoretical conversion efficiency P_{el}/P_{in} approaches 60%. Of course, it is possible to define more general functions η and β, introducing dependences on waveguide phasing, power, etc., which obviously would allow us to fit the data better. We choose not to do so because there is no very compelling theoretical reason to choose anything more complicated than a constant. The assumption here is that the data are less sensitive to the theories of absorption and upshift than to the theory of efficiency, and choosing the simplest functions provides the severest test for the efficiency theory. So, considering that there are only two adjustable parameters, there is startling agreement over the full range of the curve. In particular, the experimental data faithfully reproduce the "diode" characteristics of the theoretical formula.

The data in Fig. 1 illustrate three rf-current-drive regimes. For $u > 0$, the rf power causes the current to increase, and P_{el}/P_{rf} monotonically increases because the electric field plays an increasingly dominant role in slowing down the hot electrons. However, for large u, the Ohmic losses become large. This means that although P_{el}/P_{rf} increases, $\dot{W}/P_{rf}$ may decrease. For $u = 0$, a steady-state current is achieved and the ramp-up efficiency is, of course, zero, since the hot electrons slow down only because of collisions. For $u < 0$, the rf power is insufficient to prevent the current from decaying and, interestingly, power flows from the poloidal field into the hot electrons ($P_{el} < 0$). Nevertheless, W does not decay as rapidly as with no rf power, since the V^2/R losses are reduced. Note that, near $u = 0$, only a small electric field is present, and the theory of the steady-state ($u = 0$) rf-generated current[9,10] and the theory of rf-enhanced ($u \simeq 0$) conductivity[11] may be tested. Clearly, the experimental data here offer solid confirmation of these theories, including the scaling of the steady-state efficiency with density and phase velocity. Note that in making the comparison to the steady-state theory, we may use the second derivative of the theoretical curve in Fig. 1, which is just the so-defined steady-state efficiency, I_{rf}/P_{in} (as opposed to the ramp-up efficiency used here).

Apart from the conclusions drawn from the very small scatter in the data, what does this apparently remarkable agreement between theory and experiment tell us? While the lack of scatter indicates the identification of the relevant dimensionless parameters, the fit to the data can be used as corroboration for the theoretical relationship between these parameters. Alternatively, the theory can be assumed to be correct and the exercise of fitting the data can be used to provide rough estimates for η and β, two quantities which are difficult to measure experimentally. In making the best fit, note that η and β are not independent; e.g., if $\beta = 1$, then $\eta = 0.5$ provides the best fit, although not as good for the pair of values used in the figure. Observe that the best estimates for η and β turn out to be consistent with the expectation that large absorption requires a moderate $n_\parallel$ upshift.[13,14] The consistency of experiment and theory gives us confidence in extrapolating these results to larger machines.

There are some subtleties in the calculation of the experimental data points which we wish to elucidate. First of all, note that the ramp-up rate $\dot{I}$ varies during the rf pulse. In order to obtain the points given in Fig. 1, averages were performed over the rf pulse. The taking of averages is permissible, provided the variation is not too great, because the theoretical curve is smooth so that the average of points along some segment of the curve itself lies close to the curve. Generally, this condition was met in the PLT experiment[7] where, because the duration of the rf pulse (< 300 ms) was less than the L/R time (~ 0.5–1 s), nearly constant ramp-up rates were achieved.

The averaging of $\dot{W} - P_{ext}$ must, however, be carried out with care. This quantity is determined as a function of time

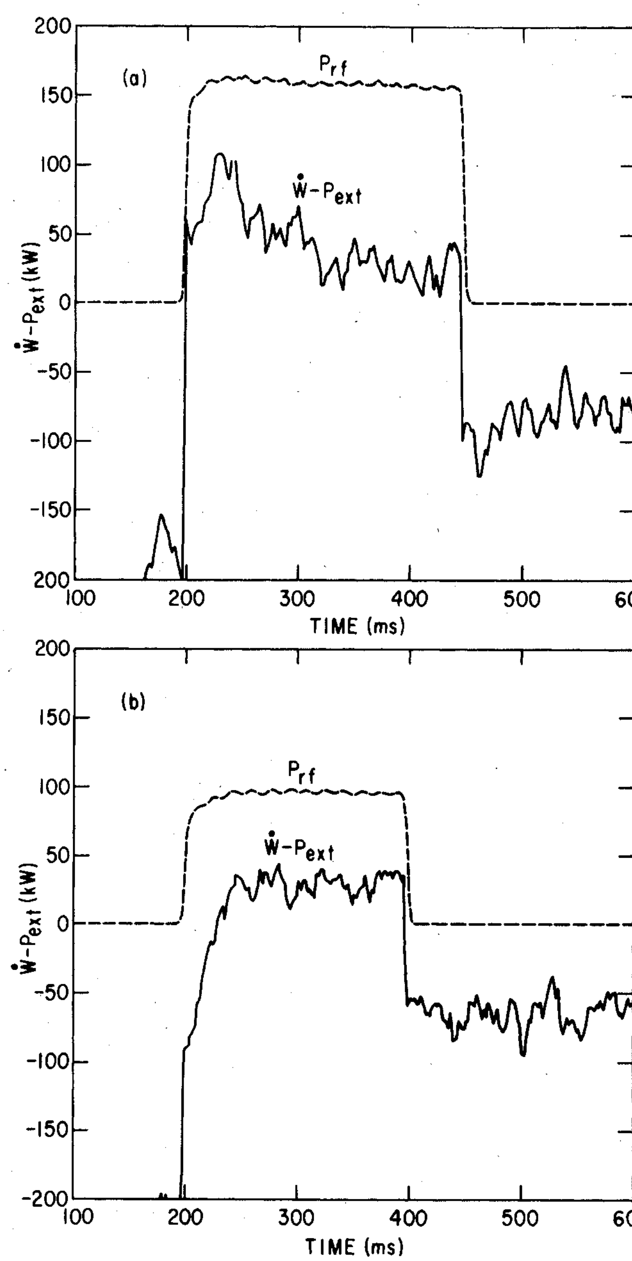

FIG. 2. Traces of $\dot{W} - P_{ext}$ vs t for two representative ramp-up shots. In (a) $P_{rf} = 160$ kW and $\dot{I} > 0$ as soon as the rf is turned on. In (b) $P_{rf} = 96$ kW and there is a delay in $\dot{I}$ changing sign.

by using the experimentally observed plasma current, an estimate of the total inductance and the currents in the external coils. The inductance includes the calculated external inductance (given by the position of the plasma) and an internal inductance, which is approximately deduced from the vertical field. Generally, $\dot{W}-P_{ext}$ is not exactly constant over the rf pulse and an average quantity is employed in Eq. (1). The question is how to perform this averaging.

Traces of $\dot{W}-P_{ext}$ for two shots are shown in Fig. 2, with Fig. 2(a) representing the more typical situation. In both cases there is a transient portion before $\dot{W}-P_{ext}$ settles down to a nearly steady value about 100 ms after the beginning of the rf pulse. Two methods of averaging $\dot{W}-P_{ext}$ suggest themselves: either averaging over the whole rf pulse or averaging over all but the first 100 ms, where $\dot{W}-P_{ext}$ is more nearly constant. The two methods give $P_{el}/P_{rf}=32\%$ and 21% for the shot shown in Fig. 2(a), and 16% and 36% for that in Fig. 2(b). When the data for all the shots are compared, the first method (which is used in Fig. 1) gives much less scatter. The scatter in data when the second method is used arises primarily from the shots represented in Fig. 2(b), which lie considerably above the other points. The reason for this may lie in the ability of the plasma to transfer energy between energetic particles and the field energy. During the initial part of the rf pulse in Fig. 2(b), the current is still decaying and there is a forward electric field. It is possible that a large population of runaways may be generated in these circumstances, and they will absorb energy from the poloidal field. However, this energy flows back into the poloidal field when $\dot{I}$ changes sign and the runaways slow down giving an apparently high efficiency.

By averaging over the whole rf pulse, we can ignore the temporary storage of energy in the runaway component that may occur in Fig. 2(b). Similarly, the initially high value of $\dot{W}-P_{ext}$ seen in Fig. 2(a) may be due to the presence of forward runaways when the rf power is first turned on. Strictly speaking we would wish to eliminate this additional source of energy when making the comparison to the theory. However, there appears to be no convenient and consistent way of doing so while still avoiding the potential problem with pulses like Fig. 2(b). If this is the explanation for the peak in Fig. 2(a), some of the experimental values for P_{el}/P_{rf} in Fig. 1 may be overestimated.

The voltage V, which is needed in the calculation of v_R and the Ohmic loss V^2/R, is computed as $(P_{ext}-\dot{W})/I$. The calculation of R is difficult. One method is to calculate the ratio of voltage to current before the rf is turned on. This method was rejected since the initial Ohmic plasma may have a significant non-Maxwellian component which lowers R. However, the non-Maxwellian or "slide-away" electrons will slow down by collisions during the rf pulse and not then dominate the conductivity. Instead R is calculated using the Spitzer-Harm conductivity with the temperature estimated using a crude energy balance and neo-Alcator scaling.[15] The temperature so obtained approximately conforms to the meager Thomson scattering data. Typically, V^2/R is a small term (about 10%) in the calculation of P_{el} and so fortunately, our results are insensitive to errors in this crude calculation of R.

We have shown in this Rapid Communication that the theory given in Ref. 8 explains the ramp-up efficiencies in PLT. The agreement is over a wide range of plasma parameters, encompassing shots in which the current is increasing, steady, and decaying. Confidence in our understanding of these experiments now allows the contemplation of larger machines utilizing the favorable rf–ramp-up regime attained in PLT. The design of such machines may use this theory[8] and recent extensions.[16] These include the circuit equations to be used to describe the evolution of the plasma over time scales comparable to or longer than the L/R time.

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[1]J. E. Stevens *et al.*, in *Proceedings of the 3rd Joint Varenna-Grenoble International Symposium on Heating in Toroidal Plasmas,* edited by C. Gormezano, G. G. Leotta, and E. Sindoni (Commission of the European Communities, Brussels, 1982), Vol. II, p. 455.

[2]S. Kubo *et al.*, Phys. Rev. Lett. **50**, 1994 (1983).

[3]F. C. Jobes *et al.*, Phys. Rev. Lett. **52**, 1005 (1984).

[4]K. Toi *et al.*, Phys. Rev. Lett. **52**, 2144 (1984).

[5]B. Lloyd *et al.*, in Proceedings of the IAEA Meeting on Noninductive Current Drive, Culham, 1983, edited by D. F. H. Start (unpublished), Vol. I, p. 250.

[6]W. M. Hooke, Plasma Phys. Controlled Fusion **26**, 133 (1984).

[7]F. C. Jobes *et al.*, Princeton Plasma Physics Laboratory Report No. PPPL-2200, 1985 (unpublished).

[8]N. J. Fisch and C. F. F. Karney, Phys. Rev. Lett. **54**, 897 (1985).

[9]N. J. Fisch, Phys. Rev. Lett. **41**, 873 (1978).

[10]N. J. Fisch and A. H. Boozer, Phys. Rev. Lett. **45**, 720 (1980).

[11]N. J. Fisch, Phys. Fluids **28**, 245 (1985).

[12]R. W. Motley *et al.*, *Plasma Physics and Controlled Nuclear Fusion Research, 1984* (IAEA, Vienna, 1985), Vol. 1, p. 473.

[13]P. T. Bonoli, R. Englade, and M. Porkolab, in *Proceedings of the Fourth International Symposium on Heating in Toroidal Plasmas,* edited by H. Knoepfel and E. Sindoni (International School of Plasma Physics, Varenna, 1984), Vol. 2, p. 1313.

[14]C. S. Liu *et al.*, Phys. Rev. Lett. **48**, 1479 (1982).

[15]B. Blackwell *et al.*, in *Plasma Physics and Controlled Nuclear Fusion Research, 1982* (IAEA, Vienna, 1983), Vol. 2, p. 27.

[16]C. F. F. Karney and N. J. Fisch, Phys. Fluids (to be published).